WHAT RESEARCH IS NEEDED IN PROBABILITY AND STATISTICS EDUCATION IN AUSTRALIA IN THE 1990s?

JANE M WATSON
University of Tasmania

The final publication of *A National Statement on Mathematics for Australian Schools* (Australian Education Council (AEC), 1991) includes Chance and Data as one of the five major content areas of study in mathematics suggested for Australian schools. Following the appearance of the *National Statement,* the Australian Association of Mathematics Teachers received money from the Department of Education, Employment and Training to fund the preparation of eight workshops for teachers to aid in the implementation of the new curriculum. One of these workshops is to introduce Chance and Data and suggest resources to be used by teachers K-12. Further, the Curriculum Corporation has initiated a national collaborative mathematics project to provide materials to bring the Chance and Data part of the mathematics curriculum alive in classrooms K to 10. For those who have been making suggestions for the inclusion of topics from probability and statistics in the curriculum for many years (e.g., Bloom & McDougall, 1988; Peard, 1990; Watson, 1980, 1989, 1991), these initiatives are whole-heartedly welcomed.

These moves reflect a worldwide effort to bring Chance and Data to every level of the mathematics curriculum. In the United States these developments are reflected in the National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) and in materials produced for teachers by the Quantitative Literacy Project for secondary schools and the Used Numbers Project for primary schools. In England and Wales, the 1991 proposal for *Mathematics in the National Curriculum* (Department of Education and the Welsh Office, 1991) devotes one of the five major Attainment Targets to Data Handling.

These initiatives have been taken in Australia, however, without the benefit of any previous educational research in this country on the learning of probability and statistics[1] and without any formal recommendations for future research on the learning and teaching of the topics or for evaluation of the efforts of preparing teachers or actually implementing the curriculum. Hence states and local schools now face the implementation of the Chance and Data part of the curriculum with no Australian research base from which to make recommendations for the preparation of teachers or for the suggestion of methods and topics realistic to the developmental level of the students. It is the purpose of this paper to suggest such a base.

OVERSEAS RESEARCH

Looking at the history of research in the area of learning probability and statistics overseas, it is clear that while progress is being made, there is still much more to accomplish. The most recent review of the research in this area is by Shaughnessy (1992). He provides an excellent summary of research associated with judgements made under uncertainty, the different approaches to research used in the area, the more meagre research associated with statistical constructs and the use of computer-based activities to reinforce them. He goes on to suggest directions for future research in seven areas: assessment, secondary students' understanding, cross-cultural studies, teachers' understanding, teaching experiments, computer software, and metacognition. The call for continued research overseas means Australian researchers have two types of opportunities available to them. There is the chance to be involved in advancing basic understanding in the area, as well as considering questions relevant to the Australian educational context. Furthermore Shaughnessy, while dealing with the sophisticated concepts which provide difficulties in learning probability and statistics, does not address adequately the problems of younger children being introduced to chance and data. This is a result of there being very little published research in the area in the previous two decades, and a delay in publishing time which means that Shaughnessy did not have access to some very recent studies.

In an earlier review of research, Hawkins and Kapadia (1984) presented a review of work done in probability up to the first International Congress on the Teaching of Statistics. They discussed the debate between followers of Piaget and Inhelder (1951) who concluded that children could not handle probability concepts until the age of formal operations, and followers of Fischbein (1975) who used an intuitive definition of probability and came to a different conclusion. Hawkins and Kapadia pointed out that much confusion has arisen as a result of researchers using different definitions of probability itself. In the area of statistics much less research has been reported and it has focused on specific constructs, like the arithmetic mean (e.g., Goodchild, 1988). One of the difficulties associated with research in this area is that much of it has been done with convenience samples of tertiary students. The applicability of the results of this research to primary and secondary students is highly suspect given the expected different levels of development present.

In their review of research on probability and statistics learning up to 1988, Garfield and Ahlgren presented the following summary, aimed at the secondary level and above.

> The literature makes it clear that far more research has been done on the psychology of probability than on other statistics concepts. In spite of this research, however, teaching a conceptual grasp of probability still appears to be a very difficult task, fraught with ambiguity and illusion. Accordingly, we make the pragmatic recommendation for two research efforts that would proceed in parallel: one that continues to explore the means to induce valid conceptions of probability, and one that explores how useful ideas of statistical inference can be taught independently of technically correct probability... As far as goals for instruction go, this view suggests a moratorium on the typical organization of statistics instruction: (a) descriptive statistics, (b) probability, and (c) inferential statistics. The intrusion of technical probability issues that are not likely to be understood will stall the learning process -- and leave a distaste that could compromise subsequent instruction as well... What is needed, however, is not debate but research (Garfield and Ahlgren, 1988, p. 57).

This discussion would indicate that secondary teachers could be facing a decade of frustration in attempting to get across to students many of the topics in the Chance and Data curriculum. For primary teachers, the situation is even less well known because as noted, virtually no research results are generally available dealing with children this young. Two groups in the United States are beginning to look at statistical understanding at lower levels. Mokros and Russell (1992) are studying concepts of average and representativeness in children in grades 4, 6 and 8, and in their teachers (Russell & Mokros, 1991). Gal, Rothchild and Wagner (1989, 1990) also are looking at statistics reasoning in children in grades 3, 6 and 9.

QUESTIONS FOR AUSTRALIA

In Australia school systems and teachers currently are implementing the Chance and Data curriculum using the best resources and advice they can get from educators and curriculum planners, all of whom are operating without the luxury of a local research base. The situation, however, does not have to stay like this throughout the decade. There is the opportunity to start now, using the National Statement as a basis, to monitor the present understandings of teachers and students, to make suggestions based on this monitoring, and to evaluate the first few years of the implementation of the curriculum. At least by the turn of the century it may be possible to use the results of research to make more precise and confident statements to help teachers and students understand and apply the concepts in the Chance and Data curriculum.

Content. The direct mathematical base for research in Australia lies in the *National Statement* itself. For Chance and Data there are 28 scope statements within the four bands which cover the years of schooling, K-12. These statements are meant to provide the framework around which systems or schools may build their Chance and Data curriculum. The following italicized disclaimer, however, indicates that the developers of the *National Statement* do not take total responsibility for getting the ordering right.

> *There is no implication that scope statements within a band should follow the sequence presented* (AEC, 1991, p. 28).

The generality of the scope statements leaves much to the teacher in deciding about the rigour with which concepts are introduced. In the first three bands which cover grades K-10, the suggested emphasis is practical and experimental rather than theoretical and formal. Examples of two of the scope statements from Band B, aimed at upper primary children, show the heavy responsibility placed on curriculum planners and teachers to provide the necessary constructs to make implementation possible, even if the emphasis is practical and experimental.

> Experiences with chance should be provided which enable children to: **B2**: for random events, systematically list possible outcomes, deduce the order of probability of outcomes and test predictions experimentally (p. 170).
>
> Experiences with data handling should be provided which enable children to: **B5**: understand what samples are, select appropriate samples from specified groups and draw informal inferences from data collected (p. 172).

Although interesting and useful activities are listed under each of these statements, assumptions are made that the teachers will know how these relate to the important concepts of randomness, probability, sampling, and inference. It is not clear that teachers currently possess these understandings themselves, let alone the ability to get them across to their students. It also is not clear whether the activities suggested are suitable for the development levels expected of the children in the Bands in which the activities are placed.

The question of how to explore teacher understanding and the appropriateness of content for children at different levels, will not be trivial. Recent overseas research offers promise however, as it has begun to move beyond the previous emphasis on specific topics, such as the arithmetic mean, to look at some of the broader concept issues involved in learning probability and statistics. Conclusions thus far have been based on studies of students, not their teachers. In Australia it will be important to look at both groups of people. The studies which offer the greatest potential as a basis for this research, have considered student beliefs about probability (e.g., Fischbein, Nello, & Marino, 1991; Garfield & delMas, 1991; Green, 1991; Konold, 1991b), conceptions of randomness (e.g., Green, 1989; Konold, Lohmeier, Pollatsek, Well, Falk, & Lipson, 1991), sampling in relation to sample size and representativeness (e.g., Gal, et al., 1989; Rubin, Bruce, & Tenney, 1991), the relationship between two variables (e.g., Konold, 1991a), the applicability of measures of central tendency (e.g., Gal, et al., 1990), and the understanding of conditional probability (e.g., Pollatsek, Well, Konold, & Hardman, 1987).

Teachers. What is needed for the rest of this decade is a concentrated effort to find out what teachers initially understand in relation to the Chance and Data curriculum and then to monitor changes as professional development takes place. It is suggested that samples of teachers be selected at the early childhood, primary, secondary, and senior secondary levels. From a relatively large sample of teachers, say 60, who are given written questionnaires to complete, a subset of 20 would be selected for in-depth interviews to explore constructs more fully. If testing took place each two years and included a monitoring of professional development activities of teachers, two outcomes would seem achievable: results of each testing would provide input for professional development activities in the next two-year period and by the end of the decade a picture of long-term development of stochastic constructs would emerge.

Students. A similar testing scheme should be developed for students, starting say with samples in Grades 3, 6 and 9. Two years later these same students would be retested and another sample chosen from Grade 3. Each sample, after the first, would be from Grades 3, 5, 8 and 11. This seems an appropriate spread, avoiding the transition from primary to secondary schooling. As with the teachers, larger samples (up to 200) would be given written questionnaires with interviews being used for a subset. The outcomes would include a monitoring of student development of understanding, as well as of the implementation of specific aspects of the Chance and Data curriculum. The results of testing should lead to suggestions for the ordering of topics in the curriculum, for the appropriateness of topics for the Bands to which they have been assigned, and for particular activities to reinforce weak concepts. Again by the end of the decade the scale of the implementation of the Chance and Data curriculum would be gauged.

Assessment. As part of this research assessment instruments would be developed for teachers and for students at various grade levels. As the emphasis on problem solving in the *National Statement* permeates the Chance and Data content area, recent research in assessment in problem solving should be applied to the development of test items. Initial testing could concentrate on two constructs which are considered essential to the successful implementation of the Chance and Data curriculum (AEC, 1991). These are (i) the conception of probability from intuitive to experimental to theoretical and (ii) the understanding of sampling from convenience to random, with associated issues of representativeness. The tasks to exhibit understanding in these areas would be selected from those suggested in overseas studies and those found to be useful in preliminary pilot studies of probabilistic and statistical reasoning.

In probability the results of Fischbein, et al. (1991) offer a feasible starting point because they look at how an intuitive understanding of sample space influences responses to classical probability tasks involving coins and dice. In the Australian context the interest first would be centred, because of the emphasis in the *National Statement* for primary children, on how an intuitive understanding influences responses to experimental probability tasks. Fischbein's items could be modified to have a frequency basis, for example, based on data presented in tables. With older students it would be possible to use Fischbein's tasks directly. In statistics Kahneman and Tversky (1972) suggest specific problems to consider the similarity of a sample to its population, the apparent randomness associated with a sample, the effect of sample size on the representativeness of a sample, and the likelihood of various samples based on population characteristics. These problems, with variations in the form of presentation and extended interviewing as suggested by Pollatsek, Konold, Well and Lima (1984), would provide a starting point for tasks. For younger children, situations could be set up providing data for the comparison of two groups performing on a given criterion, and children then asked to decide which group has performed better (e.g., Gal, et al., 1989, 1990). Tasks also would be chosen and adapted from the Statistics Reasoning Assessment instrument, currently under development in the United States (Garfield, 1991).

A THEORETICAL MODEL

Historically psychologists have played a major role in the research into the understanding of probability concepts, most notably Piaget and Inhelder (1951), Fischbein (1975) and Kahneman and Tversky (1972). Piaget contributed a developmental perspective on the performance of certain tasks, Fischbein suggested the part to be played by intuition in understanding concepts, and Kahneman and Tversky found particular difficulties, such as representativeness (Kahneman and Tversky, 1972) and availability (Tversky and Kahneman, 1973), which have been widely studied in the last decade. The developmental and multimodal functioning aspects of the SOLO Taxonomy (Biggs and Collis, 1982, 1989, 1991; Collis and Biggs, 1991) make it a natural choice for a model to use following the work of Piaget and Fischbein. The most recent products of this research stream have been in the areas of assessment of cognitive functioning during problem solving in adolescents (Collis, Watson, and Campbell, 1991), of following the development of understanding of common and decimal fractions (Watson, Collis and Campbell, 1991a, 1991b), and of developing a mapping procedure to analyse mathematical responses (Collis and Watson, 1991). Building on the work of Biggs and Collis (1982, 1989), these latest projects have specifically considered the issues of (i) multimodal functioning, (ii) the repetitive nature of the unistructural-multistructural-relational (UMR) cycle which operates within the modes of functioning, and (iii) the ability to graphically follow a response.

(i) The multimodal aspect of cognitive functioning has come to prominence in recent years due in part to the reporting of the effect of out-of-school experiences on mathematical problem solving (e.g., Resnick, 1987; Saxe, 1988). In this context the importance of intuition on people's understanding of probabilistic concepts is well documented (e.g., Fischbein, 1975; Fischbein, et al., 1991). Further the potential applicability of both probabilistic and statistical concepts outside the classroom is one of the major reasons for putting them in the mathematics curriculum (AEC, 1991). It is very likely that the intuitive and imaging aspects of the ikonic mode of functioning play a major role in solving problems associated with chance and data. If so, they should be actively developed in the methods suggested for teachers who are implementing the new curriculum. The assessment of multimodal functioning in the solving of specific probability and statistics problems would assist in making recommendations for teaching.

(ii) A second aspect of earlier research in cognitive functioning has been its focus on developmental structure. Important in this has been the observation of UMR sequences of responses to mathematical tasks set for students (Biggs and Collis, 1982). Previous work on volume measurement (Campbell, Watson and Collis, in press) and fractions and decimals (Watson, et al., 1991b) showed that the development of concepts occurred in the concrete symbolic mode of functioning via two sequential learning cycles, the first reaching the relational level by about the end of grade 6 and the second beginning there and reaching the relational level in grades 10+. It is important to explore the hypothesis that a similar developmental structure exists for concepts associated with the learning of probability and statistics. If so, it would be possible to suggest sequencing of the curriculum to aid learning.

(iii) Finally, the mapping procedure to analyze responses has helped to graphically portray the cognitive functioning taking place during problem solving (Chick, Watson and Collis, 1988; Collis and Watson, 1991). The procedure has been used to study error patterns in area problems (Watson, Chick and Collis, 1988), to explore strategies for problem solving (Watson, in press), to suggest how the transition from the ikonic to concrete symbolic mode occurs (Watson and Mulligan, 1990), and to observe perseverance patterns during problem solving (Taplin, 1991). There is potential for all of these applications to be made in relation to topics in the Chance and Data curriculum.

It is essential to have a theoretical basis from which to analyse student and teacher understanding of constructs in probability and statistics. There are other models other than the SOLO Taxonomy which also provide potential for research, for example those based on information processing theories (e.g., Scholz, 1991). In terms of potential communication of results with the wider education community, however, it is felt that a developmental model is likely to be the most useful. This is especially true for research covering a large part of the K-12 curriculum. A stage-based model fits well with the types of conceptions of stochastics suggested by Shaughnessy (1992):

1. *Non-statistical.* Indicators: responses based on beliefs, deterministic models, causality, or single outcome expectations;

2. *Naive-statistical.* Indicators: use of judgmental heuristics, such as representativeness, availability, anchoring, balancing;

3. *Emergent-statistical.* Indicators: ability to apply normative models to simple problems; recognition that there is a difference between intuitive beliefs and a mathematized model; perhaps some training in probability and statistics;

4. *Pragmatic-statistical.* Indicators: an in-depth understanding of mathematical models of chance (i.e. frequentist, classical, Bayesian); ability to compare and contrast various models of chance; ... recognition of the limitations of and assumptions of various models (p.485).

Those familiar with the SOLO Taxonomy would likely be able to parallel the four categories with their understanding of unistructural, multistructural, relational, and extended abstract levels within the concrete symbolic mode.

IMPLICATIONS OF THE RESEARCH

The significance of the suggested research lies in providing a fundamental structure for a new area of the school curriculum and in monitoring its implementation in the early years. It also is possible, however, to speculate on some of the implications of such research for the classroom. Traditionally the limited exposure to topics in probability and statistics which has been present in the mathematics curriculum resulted in the introduction of topics which were then not developed or applied beyond their unistructural acquisition as constructs. The most notable example of this is the introduction of the arithmetic mean to children in the upper primary school. Most children acquire an 'add-em-up-and-divide' algorithm but no understanding of how the mean can be used to make decisions about data sets. The same may be said for the basic definitions in probability which are introduced in high school. Very recently the move to increase the presence of probability and statistics in the mathematics curriculum in the United States (NCTM, 1989) has produced several curriculum projects aimed at increasing the level of sophistication at which the topics are learned in schools. One project, Used Numbers, written for grades K to 6, shows a definite expectation that students will work at higher cognitive levels as they progress through the curriculum (see e.g., Russell & Stone, 1990; Corwin & Friel, 1990). Although not explicitly stated in psychological terms, it is possible to show how the suggested work in each of the six books in the Used Number series, progresses from unistructural to multistructural to relational in terms of the cognitive levels required to complete the tasks set. As well, the tasks require considerable interaction with the world outside formal mathematics and hence also offer the opportunity to explore and develop multimodal functioning.

There is great potential for interaction between the theoretical cognitive research base and the suggestions made in the Used Number project curriculum. It is likely that the materials would form the basis for interview tasks, as well as being suggested for teacher use as the curriculum is introduced. This would make possible the further evaluation of the developmental levels of responses of children as implementation occurs.

Finally, as the probability and statistics component of the mathematics curriculum can be argued to be the part most closely related to out-of-school experiences, the intuitive/ikonic involvement may be an aspect which is encouraged as a result of data analysis. This is a different emphasis than is commonly associated with topics in the traditional mathematics curriculum. It would be of great interest for teaching methodology to find out the influence of the ikonic mode on understanding of and problem solving in relation to the topics in the Chance and Data curriculum.

CONCLUSION

This paper has argued that there is an urgent need for research into the understanding of concepts related to probability and statistics to be carried out in Australia in the wake of the publication of *A National Statement on Mathematics for Australian Schools* with its emphasis on Chance and Data. The foundations suggested for this research are found in developmental psychology and in work beginning to be done in the area by mathematics educators overseas. The research would assess responses to tasks in probability and statistics in order to (i) build a cognitive model of student and teacher understanding of the topics, (ii) make suggestions for the introduction and ordering of these new topics in the mathematics curriculum and (iii) provide assessment procedures for the evaluation of the implementation of the curriculum.

FOOTNOTE

[1]The only papers which the author could find reporting Australian research in stochastics were written by Peard (1991) on a study considering misconceptions in probability in samples of gamblers and non-gamblers, and by Truran (1991) on children's comparison of probabilities in Grades 4, 6, 8 and 10. They appeared after the *National Statement* was complete.

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